What Difference Does A Distribution Make?

The thought occurred to me to explore the importance of selecting the right probability distribution as you create your risk model. Is it of major importance? Our task is to find the best distribution to portray the shape of an uncertain variable. I will present the case and give you my opinion, but, in the end, you are encouraged to reach your own conclusions.

First, some background information.

A probability distribution is a mathematical method for describing the uncertainty of a variable. There are many forms and types of probability distributions, each of which describes a range of possible values and their probabilities of occurrence.

Risk analysis is a quantitative method that seeks to determine the probability distributions of the outcomes resulting from decisions. In general, the techniques in risk analysis encompass four steps:

> **1. Developing a Model. Defining your problem or situation in an Excel model.**
> **2. Identifying Inputs and Outputs. Determining uncertain inputs in your model and specifying their possible values with probability distributions and identifying the important outputs you want to analyze.**
> **3. Analyzing the Model with Simulation. Running many scenarios, each with sampled values for the uncertain inputs, to determine the probability distributions of your outputs.**
> **4. Making a Decision. Using the simulated results and personal preferences to make informed choices.**

Inputs to the model may be certain or uncertain. A certain, or deterministic input, is a value that you know and about which there is no uncertainty. An uncertain input requires an entry as to the nature of the variable's uncertainty. Among other things,

Jerry Scherer, President
JB Scherer Consulting Group LLC
PO Box 289
Glencoe, IL 60022-0289
www.jbschererconsultinggroup.com
januerry@comcast.net
(847) 835-1175 (Phone)
(847) 835-1189 (Fax)
(708) 805-3300 (Mobile)

this will encompass the shape of the uncertainty. Does it have a "bell shape" [Normal]? Is it triangular [Triangular or Pert]? Does it simply have a minimum and a maximum value with an equal likelihood that any value within that range may occur [Uniform]? Is the data that you are sampling continuous or discrete? A discrete distribution is one where only integer values will be returned. There cannot be partial answers. A continuous distribution can take on any value in a range.

There is much to be considered as you examine the raw data that will find its way into your model. In our case, the most uncertain of that data is the volume of product that we will sell. The Marketing Department has presented us with a range of values for next year's volume and our task is to find the best probability distribution to portray the shape of that data set.

Minimum	**$50,000**
Most Likely	**$60,000**
Maximum	**$130,000**

We will consider three possible distributions and then compare their outcomes.

<u>PERT Distribution</u>

What we know about this variable is that there is no available data (other than the Marketing Department forecast). Since we have a reasonable guess for minimum and maximum values, it is appropriate to consider the use of a PERT distribution, one of numerous choices in the family of continuous distributions. An interesting sidebar about the PERT distribution is that its mean is equal to the weighted average of its minimum, most likely and maximum values with four times the weight applied to the most likely value.

<u>TRIANGULAR Distribution</u>

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januerry@comcast.net
(847) 835-1175 (Phone)
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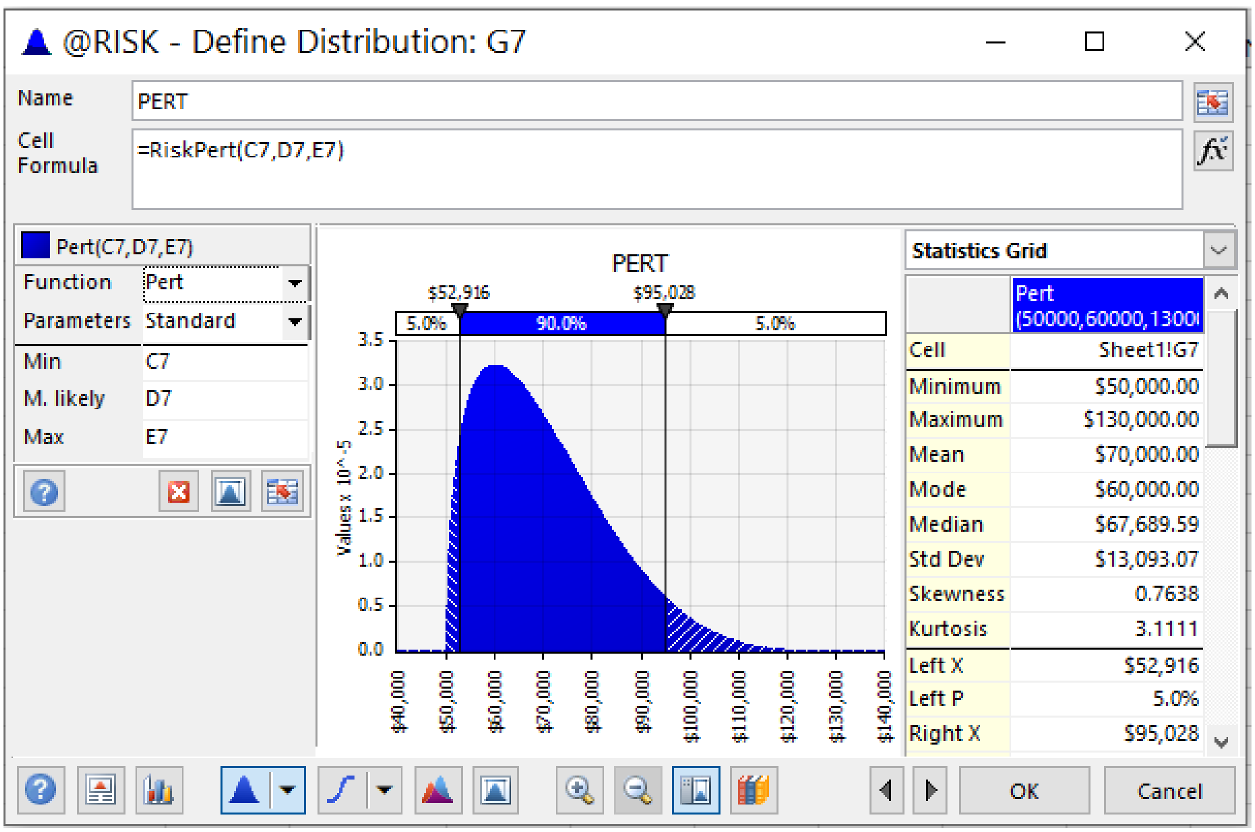
Since we have minimum, most likely and maximum values, it would also be reasonable to consider the use of a TRIANGULAR distribution. Like PERT, it considers the same values, but its mean is simply the weighted average of the minimum, most likely and maximum values.

<u>UNIFORM Distribution</u>

Sometimes referred to as the "no clue distribution," the UNIFORM distribution requires only a minimum and maximum value and posits that any value in between those extremes has an equal likelihood of occurrence.

Results of Pert Distribution



What can we learn from this PERT Distribution? 90% of the results fall between $52,916 and $95,038. The 5% tail on the low

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januerry@comcast.net
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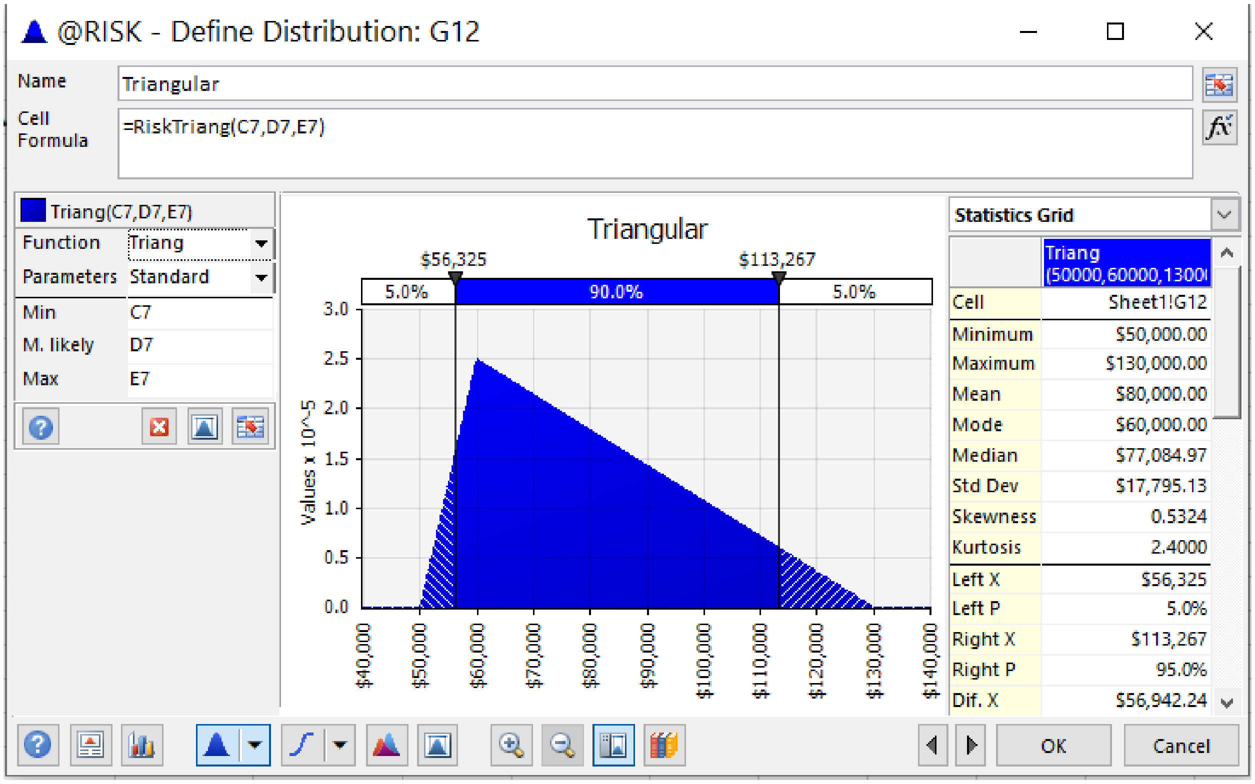
side is equal to $2,916 and the 5% tail on the high side is equal to $34,972.

Now let us look at the TRIANGULAR Distribution using the same parameters as the PERT Distribution.



Note that 90% of the results fall between $56,325 and $113,267. Thus, the tails at the lower and upper ends are $6,325 and $16,733, respectively.

If we overlay the PERT Distribution on the TRIANGULAR Distribution and move the slider on the right hand side to $90,000 so that both distributions are evaluating the same data set:

Jerry Scherer, President
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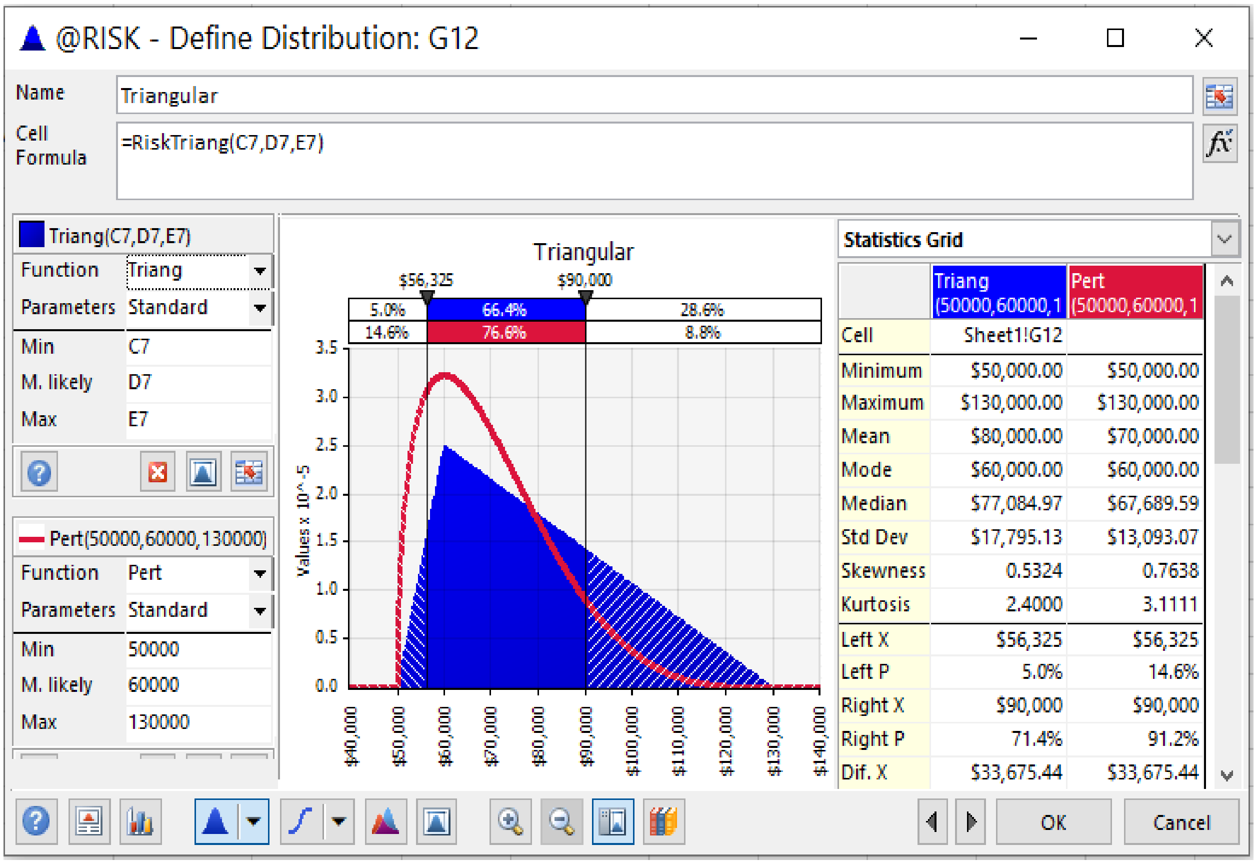

Note that beyond the $90,000 mark, there are three times as many TRIANGULAR distribution results versus those of the PERT distribution. Namely 8.8% of the PERT results can be found whereas 28.6% of the TRIANGULAR results are there.

While the arguments for both distributions are identical, the PERT suggests that more of the results will fall within the most likely range. Thus, there are fewer tail values.

Finally, let us look at the results of the UNIFORM distribution.

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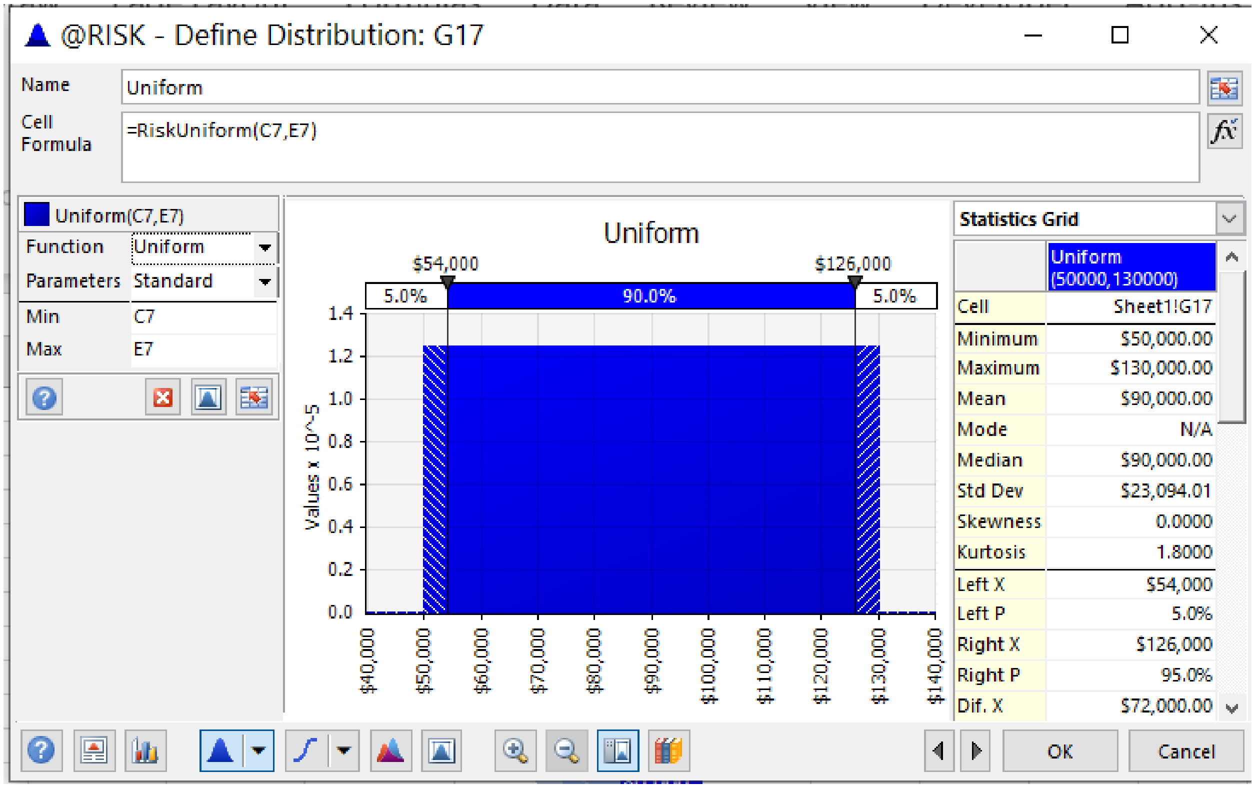

Note that 90% of the results fall within the range from $54,000 to $126,000 and the tails at both extremes are $4,000.

Overlaying the other distributions so that we can compare all three, you will note that 50% of the UNIFORM distribution values fall beyond the $90,000 level.

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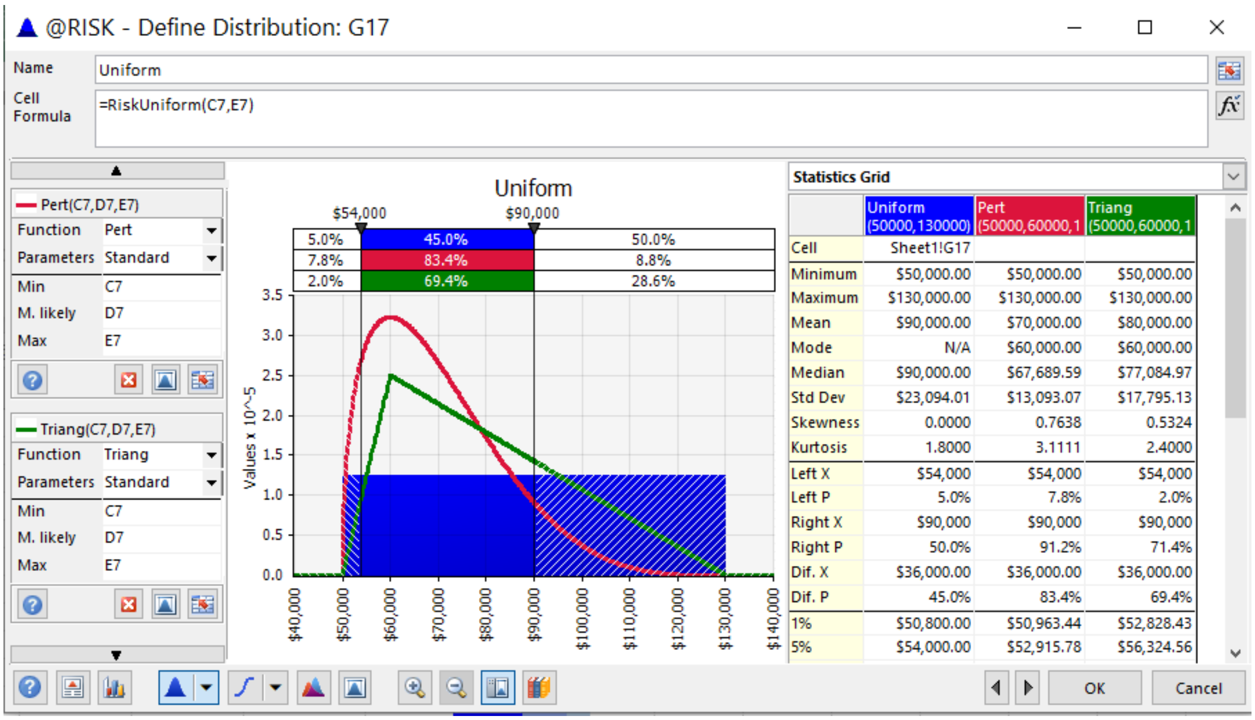

SUMMARY MEASURES OF VOLUME

	PERT	TRIANGULAR	UNIFORM
Mean	**$70,000**	**$ 80,000**	**$ 90,000**
Standard Deviation	**$13,093**	**$17,795**	**$23,064**
5th percentile	**$52,916**	**$56,325**	**$54,000**
95th percentile	**$95,028**	**$113,267**	**$126,000**

I think the evidence is clear. It is quite important to understand the shape of the distribution that will best fit your dataset. If you have historical data, use a distribution fitting tool to ascertain the appropriate distribution. In the absence of data, begin to collect data for future reference, and, in the meantime try one of the more popular distributions to get started.

The important lesson to be learned is to get started. Decision making should not be based on hunches or gut feelings. Mathematical tools may be brought into play to support those intuitive feelings before committing precious company resources.

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